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AERODYNAMIC COEFFICIENTS FOR CONTROL SURFACE-TAB COUPLING IN SUBSONIC, BIDIMENSIONAL, UNSTEADY FLOW

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AERODYNAMIC COEFFICIENTS FOR CONTROL SURFACE-TAB COUPLING IN SUBSONIC, BIDIMENSIONAL, UNSTEADY FLOW

Rolland Dat and Jean-Louis Meurzec

ABSTRACT. An approximate method of calculation of the aerodynamic coefficients in question is presented and the results are compared with figures from tables based on an exact theory. The results are found to be extremely close in incompressible flow and satisfactory, though more divergent, for compressible flow.

I. Introduction

The method of independent sections is still used for the determination of/60* the generalized aerodynamic forces which occur in aircraft flow calculations. The wing being studied is divided into sections parallel to the wind, and it is assumed that the flow is bidimensional in each of these sections.

This method is justified for lifting surfaces of great aspect ratio, and the economic advantages offered by it are considerable.

In practical applications, tables are used giving the coefficients of the matrices which relate the position parameters of a section possessing a control surface and the corresponding force components (Figure 1) by a matrix relationship which is valid for a harmonic oscillatory motion.

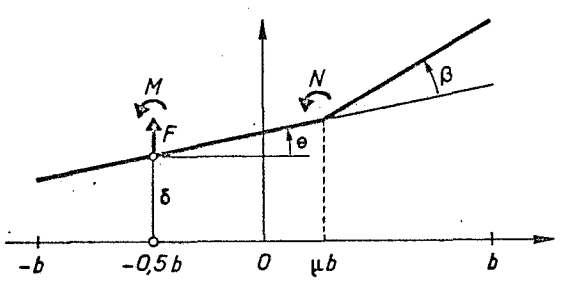

Fig. 1.

$$\begin{bmatrix} Fb \\ M \\ N \end{bmatrix} = -\pi\rho V^2 b^2 l A_{III}(k, M, \mu) \begin{bmatrix} \delta/b \\ 0 \\ \beta \end{bmatrix}$$

(l = width of the section)
($2b$ = chord of the section).

*Numbers in the margin indicate pagination in the foreign text.

A_{III} is generally put in the form $A_{III} = A'_{III} + ikA'''_{III}$, where k is the reduced pulsation: $k = \omega b/V$ (ω being the pulsation).

The matrices A'_I and A'''_{III} are dependent on k, the Mach number, and the reduced abscissa of the hinge, μ. (The order III of the matrix is indicated by a subscript in order to avoid any confusion later.)

The coefficients of these matrices are ordinarily designated by the following notations:

$$A'_{III} = \begin{bmatrix} k'_a & k'_b & k'_d \\ m'_a & m'_b & m'_d \\ n'_a & n'_b & n'_d \end{bmatrix} \qquad A'''_{III} = \begin{bmatrix} k'''_a & k'''_b & k'''_d \\ m'''_a & m'''_b & m'''_d \\ n'''_a & n'''_b & n'''_d \end{bmatrix}.$$

For a wing section which has a control surface and a tab one more parameter is needed, and matrix A_{III} has to be completed with a fourth line and a fourth column. The conventions adopted in this case are indicated in Figure 2.

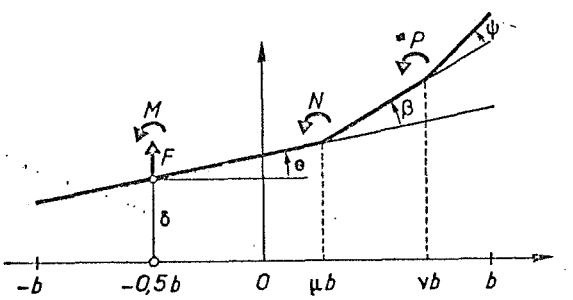

Fig. 2.

We have

$$\begin{bmatrix} Fb \\ M \\ N \\ P \end{bmatrix} = -\pi\rho V^2 b^2 l A_{IV}(k, M, \mu, \nu) \begin{bmatrix} \delta/b \\ 0 \\ \beta \\ \psi \end{bmatrix}$$

with

$$A_{IV} = A'_{IV} + ikA'''_{IV}$$

$$A'_{IV} = \begin{bmatrix} k'_a & k'_b & k'_d & k'_t \\ m'_a & m'_b & m'_d & m'_t \\ n'_a & n'_b & n'_d & n'_t \\ p'_a & p'_b & p'_d & p'_t \end{bmatrix} \qquad A'''_{IV} = \begin{bmatrix} k'''_a & k'''_b & k'''_d & k'''_t \\ m'''_a & m'''_b & m'''_d & m'''_t \\ n'''_a & n'''_b & n'''_d & n'''_t \\ p'''_a & p'''_b & p'''_d & p'''_t \end{bmatrix}.$$

The matrix of aerodynamic coefficients A_{IV} is dependent on k, the Mach number M of the reduced abscissa of the hinge of the control survace, μ, and the reduced abscissa of the hinge of the tab ν.

Under these conditions, complete tables of coefficients would occupy a con-

siderable volume and would not be very convenient to use.

Let us note that the greater part of the coefficients of matrix A_{IV} are contained in matrix A_{III}, since the pressure induced by the deflection of a tab does not differ from that induced by a control surface whose hinge has the abscissa ν, and we have: /61

$$p_a(\nu) = n_a(\nu), \quad p_b(\nu) = n_b(\nu), \quad p_t(\nu) = n_d(\nu), m_t(\nu) = m_d(\nu), \quad k_t(\nu) = k_d(\nu).$$

On the other hand, the effect of the control surface on the tab (p_d) and of the tab on the control surface (n_t) are dependent on μ and ν and are to be computed separately.

Two solutions are conceivable:

1) To program the theory of bidimensional flow and redo the calculation of the aerodynamic coefficients for each application, or

2) To write that the pressure induced by the control surface or the tab is a superposition of known functions, and determine the coefficients of the superposition by solving, using the results which figure in matrices A_{III}.

The first solution leads to complex calculations for $M \neq 0$, and the second is preferable, even though it is only approximate.

II. Principle of the Method

Let $c_d(x)$ be the coefficent of pressure induced by the oscillation of the control surface, and $c_t(x)$ the pressure induced by the oscillation of the tab.

c_d and c_t break down into a real part and an imaginary part as follows:

$$c_d = c'_d + ik\,c''_d$$
$$c_t = c'_t + ik\,c''_t.$$

Then, let $h_1, \ldots, h_4$ be the functions that define the degrees of freedom of the section:

h_1 = 1 (translation)

h_2 = 0.5 + x (oscillation around the axis of reference)

$h_3 = \begin{cases} x - \mu \text{ for } x > \mu \text{ (oscillation of the control surface)}, \\ o \text{ for } x < \mu, \end{cases}$

$h_4 = \begin{cases} x - \nu \text{ for } x > \nu \text{ (oscillation of the tab)}, \\ o \text{ for } x < \nu. \end{cases}$

The coefficients of the matrix A_{IV} are the scalar products of the functions h by the functions c, and the last two columns are given by:

$$\begin{bmatrix} k'_d \\ m'_d \\ n'_d \\ p'_d \end{bmatrix} = \begin{bmatrix} \bar{h}_1 c'_d \\ \bar{h}_2 c'_d \\ \bar{h}_3 c'_d \\ \bar{h}_4 c'_d \end{bmatrix} \qquad \begin{bmatrix} k'_t \\ m'_t \\ n'_t \\ p'_t \end{bmatrix} = \begin{bmatrix} \bar{h}_1 c'_t \\ \bar{h}_2 c'_t \\ \bar{h}_3 c'_t \\ \bar{h}_4 c'_t \end{bmatrix} \tag{1}$$

where the notation $\bar{h}c$ represents the scalar product of the functions:

$$\bar{h}c = \int_{-1}^{1} h(x)c(x)dx.$$

The coefficients k''', m''', ... are obtained by replacing c' with c'''.

If we consider the coefficients that are dependent on c'_d, we note that three of them are known (k'_d, m'_d, and n'_d), and that the fourth (p'_d) is one of the coefficients sought. Under these conditions we may represent c'_d by a superposition of three base functions and determine the three superposition coefficients by solving. It will then remain to carry out the computation of p'_d.

The same process can be applied to the coefficients that depend on c'_t, c'_d, and c''_t.

The representation of the functions c in one base is written:

$$\begin{aligned} c'_d &= \left[F_1(\mu, x)\ F_2(\mu, x)\ F_3(\mu, x)\right] \begin{bmatrix} X_{1d} \\ X_{2d} \\ X_{3d} \end{bmatrix}, \\ c'''_d &= \left[G_1(\mu, x)\ G_2(\mu, x)\ G_3(\mu, x)\right] \begin{bmatrix} Y_{1d} \\ Y_{2d} \\ Y_{3d} \end{bmatrix} \\ c'_t &= \left[F_1(\nu, x)\ F_2(\nu, x)\ F_3(\nu, x)\right] \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix}, \\ c'''_t &= \left[G_1(\nu, x)\ G_2(\nu, x)\ G_3(\nu, x)\right] \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ Y_{3t} \end{bmatrix}. \end{aligned} \tag{2}$$

The functions F, ..., G should be chosen judiciously. In particular, they should contain the peculiarities of the pressure on the profile. In practice, we shall use the solution of the bidimensional problem, which is known, and choose among the functions that appear in that solution the ones that are most meaningful.

A computer program has been developed with the following functions:

$$F_1(\mu, x) = \sqrt{\frac{1-x}{1+x}} \qquad G_1(\mu, x) = \sqrt{\frac{1-x}{1+x}}$$

$$F_2(\mu, x) = (x-\mu) \wedge (x, \mu) \qquad G_2(\mu, x) = x\sqrt{\frac{1-x}{1+x}}$$

$$F_3(\mu, x) = \wedge (x, \mu) \qquad G_3(\mu, x) = (x-\mu) \wedge (x, \mu)$$

$$\text{with } \wedge (x, \mu) = \frac{1}{2} L \left| \frac{1-x\mu + \sqrt{1-\mu^2}\ \sqrt{1-x^2}}{1-x\mu - \sqrt{1-\mu^2}\ \sqrt{1-x^2}} \right|.$$

The calculation then becomes simple. By substituting the relationships of (2) in (1), we get

$$\begin{bmatrix} k'_d \\ m'_d \\ n'_d \\ p'_d \end{bmatrix} = \begin{bmatrix} \bar{h}_1 F_1(\mu) & \bar{h}_1 F_2(\mu) & \bar{h}_1 F_3(\mu) \\ \bar{h}_2 F_1(\mu) & \bar{h}_2 F_2(\mu) & \bar{h}_2 F_3(\mu) \\ \bar{h}_3 F_1(\mu) & \bar{h}_3 F_2(\mu) & \bar{h}_3 F_3(\mu) \\ \bar{h}_4 F_1(\mu) & \bar{h}_4 F_3(\mu) & \bar{h}_4 F_3(\mu) \end{bmatrix} \begin{bmatrix} X_{1d} \\ X_{2d} \\ X_{3d} \end{bmatrix}$$

$$\begin{bmatrix} k'_t \\ m'_t \\ n'_t \\ p'_t \end{bmatrix} = \begin{bmatrix} \bar{h}_1 F_1(\nu) & \bar{h}_1 F_2(\nu) & \bar{h}_1 F_3(\nu) \\ \bar{h}_2 F_1(\nu) & \bar{h}_2 F_2(\nu) & \bar{h}_2 F_3(\nu) \\ \bar{h}_3 F_1(\nu) & \bar{h}_3 F_2(\nu) & \bar{h}_3 F_3(\nu) \\ \bar{h}_4 F_1(\nu) & \bar{h}_4 F_2(\nu) & \bar{h}_4 F_3(\nu) \end{bmatrix} \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix}$$

and analogous formulas for the coefficients ($''$).

By solving, we get:

$$p'_d = \left[\bar{h}_4 F_1(\mu)\ \bar{h}_4 F_2(\mu)\ \bar{h}_4 F_3(\mu)\right] \begin{bmatrix} h_1 F_1(\mu) & h_1 F_2(\mu) & h_1 F_3(\mu) \\ h_2 F_1(\mu) & h_2 F_2(\mu) & h_2 F_3(\mu) \\ h_3 F_1(\mu) & h_3 F_2(\mu) & h_3 F_3(\mu) \end{bmatrix} \begin{bmatrix} k'_d \\ m'_d \\ n'_d \end{bmatrix}$$

$$n'_t = \left[\bar{h}_3 F_1(\nu)\ \bar{h}_3 F_2(\nu)\ \bar{h}_3 F_3(\nu)\right] \begin{bmatrix} h_1 F_1(\nu) & h_1 F_2(\nu) & h_1 F_3(\nu) \\ h_2 F_1(\nu) & h_2 F_2(\nu) & h_2 F_3(\nu) \\ h_4 F_1(\nu) & h_4 F_2(\nu) & h_4 F_3(\nu) \end{bmatrix} \begin{bmatrix} k'_t \\ m'_t \\ p'_t \end{bmatrix}.$$

The functions giving p'''_d and m'''_d are obtained by substituting the functions G for the functions F, and the coefficients k''' ... p''' for k', ..., p'. /62

III. Results

By applying this method, calculations have been done for various values of the pair μ, ν and of the reduced pulsation k.

The results of these calculations have been compared with the coefficients derived by the exact theory:

1) For incompressible flow [1];

2) For compressible flow at M = 0.7 [2]. In this case the elements of comparison were relatively reduced.

The tables below give the results of the approximate calculation and those

which come from the tables, for each of the coefficients p'_d, p'''_d, n'_t, n'''_t, in the case of incompressible flow and that of compressible flow.

TABLE I. INCOMPRESSIBLE FLOW*

		$\mu = 0{,}70$	$\nu = 0{,}88$	$\mu = 0{,}60$	$\nu = 0{,}88$	$\mu = 0{,}60$	$\nu = 0{,}84$
$M = 0$	k	0,1	2	0,1	2	0,1	2
p'_d	approx. cal.	0,00108	0,00099	0,00093	0,00083	0,00194	0,00174
	tables	0,00108	0,00099	0,00093	0,00083	0,00193	0,00174
p'''_d	approx. cal.	0,00006	0,00215	0,00071	0,00125	0,00014	0,00504
	tables	0,00006	0,00215	0,00071	0,00125	0,00014	0,00504
n'_t	approx. cal.	0,01966	0,01909	0,03438	0,03314	0,03511	0,03371
	tables	0,01968	0,01906	0,03441	0,03318	0,03510	0,03371
n'''_t	approx. cal.	0,00005	0,00737	— 0,00053	0,00601	0,00012	0,01734
	tables	0,00005	0,00737	— 0,00054	0,00601	0,00012	0,01734

TABLE II*

		$\mu = 0{,}40$	$\nu = 0{,}88$	$\mu = 0{,}40$	$\nu = 0{,}80$	$\mu = 0$	$\nu = 0{,}80$
$M = 0$	k	0,1	2	0,1	2	0,1	2
p'_d	approx. cal.	0,00076	0,00065	0,00280	0,00239	0,00224	0,00179
	tables	0,00076	0,00065	0,00280	0,00239	0,00224	0,00179
p'''_d	approx. cal.	0,00090	0,00153	0,00309	0,00536	0,00419	0,00683
	tables	0,00090	0,00153	0,00309	0,00536	0,00419	0,00683
n'_t	approx. cal.	0,07138	0,06786	0,07889	0,07459	0,20108	0,18463
	tables	0,07139	0,06793	0,07893	0,07458	0,20115	0,18462
n'''_t	approx. cal.	— 0,00695	0,01156	— 0,00034	0,04589	— 0,03723	0,05157
	tables	— 0,00696	0,01156	— 0,00055	0,04589	— 0,03723	0,05157

TABLE III. COMPRESSIBLE FLOW*

		$\mu = 0{,}80$	$\nu = 0{,}88$	$\mu = 0{,}40$	$\nu = 0{,}88$	$\mu = 0{,}40$	$\nu = 0{,}80$
$M = 0{,}7$	k	0,1	0,7	0,1	0,7	0,1	0,7
p'_d	approx. cal.	0,00187	0,00192	0,00101	0,00112	0,00370	0,00411
	tables	0,00186	0,00191	0,00101	0,00108	0,00372	0,00400
p'''_d	approx. cal.	0,00060	0,00109	0,00123	0,00206	0,00411	0,00712
	tables	0,00060	0,00104	0,00123	0,00206	0,00410	0,00714
n'_t	approx. cal.	0,01127	0,01144	0,09867	0,09547	0,10884	0,10914
	tables	0,01123	0,01137	0,09838	0,09824	0,10854	0,11044
n'''_t	approx. cal.	0,00052	0,00180	— 0,02656	— 0,00665	— 0,01909	0,00759
	tables	0,00036	0,00157	— 0,02875	— 0,00807	— 0,02002	0,00665

*Commas indicate decimal points.

TABLE IV*

		$\mu = 0$	$\nu = 0.88$	$\mu = 0$	$\nu = 0,80$	$\mu = 0$	$\nu = 0,40$
M = 0.7	k	0.1	0.7	0.1	0.7	0.1	0.7
p'_d	approx. cal.	0,00080	0,00103	0,00290	0,00372	0,04874	0,06221
	tables	0,00080	0,00081	0,00289	0,00316	0,04878	0,05987
p'''_d	approx. cal.	0,00159	0,00267	0,00546	0,00938	0,06519	0,12994
	tables	0,00160	0,00273	0,00550	0,00962	0,06532	0,13379
n'_t	approx. cal.	0,23165	0,21180	0,27382	0,26122	0,28608	0,32656
	tables	0,23109	0,22065	0,27349	0,26904	0,28470	0,32022
n'''_t	approx. cal.	— 0,13576	— 0,05398	— 0,13822	— 0,03183	0,00822	0,19473
	tables	— 0,14375	— 0,05824	— 0,14309	— 0,03501	0,00840	0,19570

*Commas indicate decimal points.

Examination of the tables shows perfect agreement between the exact theory/63 and the approximate calculation in the case of incompressible flow. In compressible flow the results are similarly satisfactory; significant relative deviations are found only for the coefficient n'''_t when it becomes very small in absolute value.

REFERENCES

1. Küssner, H.G. and H. Gollnitz, *Tabellen der Aerodynamischen Derivativa des Schwingenden Streckenprofils mit Knicken und Stufen. DLR FB 64-05*, May 1964.
2. Minhinnick, I.T., *Subsonic Aerodynamic Flutter Derivatives for Urings and Control Surfaces. RAE Report No.: Structures 87*, with corrigenda and addenda.

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